Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced

Thursday 24 January 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A, the length of the string is 0.56 m.

(*a*) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(*b*) Find the angle θ.

(3)

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2. A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal table. When P is a distance x metres from a fixed point O on the line, it experiences a force of magnitude $\frac{16}{5x^2}$ N away from O in the direction OP. Initially P is at a point 2m from O and is moving towards O with speed 8 m s^{-1}.

Find the distance of P from O when P first comes to rest.

(8)

3.

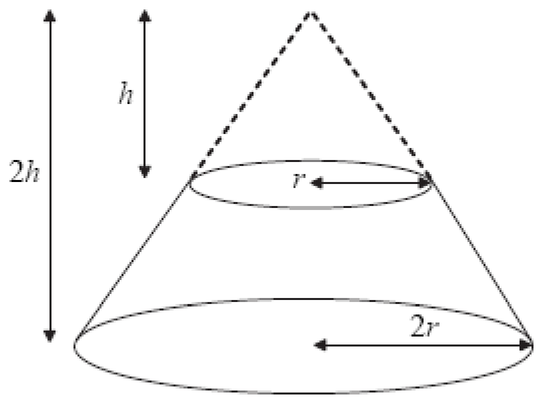

Figure 1

A uniform solid S is formed by taking a uniform solid right circular cone, of base radius $2r$ and height $2h$, and removing the cone, with base radius r and height h, which has the same vertex as the original cone, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of S from its larger plane face is $\frac{11}{28}h$.

(5)

The solid S lies with its larger plane face on a rough table which is inclined at an angle $\theta°$ to the horizontal. The table is sufficiently rough to prevent S from slipping.

Given that $h = 2r$,

(*b*) find the greatest value of θ for which S does not topple.

(3)

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4. A particle P of mass m lies on a smooth plane inclined at an angle 30° to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(*a*) find V in terms of a and g,

(6)

(*b*) find, in terms of a and g, the speed of P when the string first becomes slack.

(4)

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5. A car of mass m moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1}.

Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of mg, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

6.

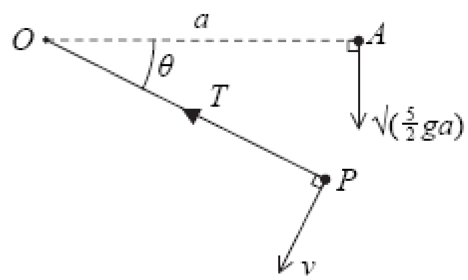


Figure 2

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. At time $t = 0$, P is projected vertically downwards with speed $\sqrt{(\frac{5}{2}ga)}$ from a point A which is at the same level as O and a distance a from O. When the string has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Figure 2.

(*a*) Show that $v^2 = \dfrac{ga}{2}(5 + 4\sin\theta)$.

(3)

(*b*) Find T in terms of m, g and θ.

(3)

The string becomes slack when $\theta = \alpha$.

(*c*) Find the value of α.

(3)

The particle is projected again from A with the same velocity as before. When P is at the same level as O for the first time after leaving A, the string meets a small smooth peg B which has been fixed at a distance $\frac{1}{2}a$ from O. The particle now moves on an arc of a circle centre B. Given that the particle reaches the point C, which is $\frac{1}{2}a$ vertically above the point B, without the string going slack,

(*d*) find the tension in the string when P is at the point C.

(6)

7. A particle P of mass 2 kg is attached to one end of a light elastic string, of natural length 1 m and modulus of elasticity 98 N. The other end of the string is attached to a fixed point A. When P hangs freely below A in equilibrium, P is at the point E, 1.2 m below A. The particle is now pulled down to a point B which is 0.4 m vertically below E and released from rest.

(*a*) Prove that, while the string is taut, P moves with simple harmonic motion about E with period $\frac{2\pi}{7}$ s.

(5)

(*b*) Find the greatest magnitude of the acceleration of P while the string is taut.

(1)

(*c*) Find the speed of P when the string first becomes slack.

(3)

(*d*) Find, to 3 significant figures, the time taken, from release, for P to return to B for the first time.

(7)

TOTAL FOR PAPER: 75 MARKS

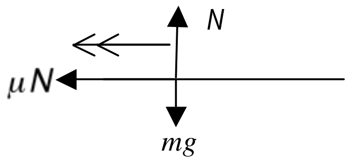
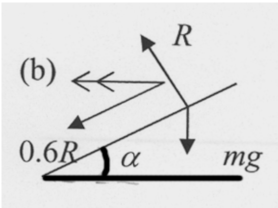
END

January 2008
6679 Mechanics Mathematics
M3 Mark Scheme

Question Number	Scheme	Marks
1.(a)	T or $\frac{\lambda \times e}{l} = mg$ (even T=m is M1, A0, A0 sp case)	M1
	$\frac{\lambda \times 0.16}{0.4} = 2g$	A1
	$\Rightarrow \lambda = \underline{49\text{ N}}$ or 5g	A1 (3)
(b)	Special case $T \sin\theta = mg$ giving θ = 30 is M1 A0 A0 unless there is evidence that they think θ is with horizontal – then M1 A1 A0	
	R(↑) $T\cos\theta = mg$ or $\cos\theta = \frac{mg}{T}$	M1
	$49\frac{0.32}{0.4}.\cos\theta = 19.6$ or $4g.\cos\theta = 2g$ or $2mg.\cos\theta = mg$ (ft on their λ)	A1ft
	$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$ (or $\frac{\pi}{3}$ radians)	A1 (3)
		6
2. a)	m 'a' = $\pm\frac{16}{5x^2}$, with acceleration in any form (e.g. $\frac{d^2x}{dt^2}$, $v\frac{dv}{dx}$, $\frac{dv}{dt}$ or	B1
	Uses $a = v\frac{dv}{dx}$ to obtain $k v\frac{dv}{dx} = \pm k'\frac{32}{x^2}$	M1
	Separates variables, $k\int v\,dv = k'\int \frac{32}{x^2}dx$	dM1
	Obtains $\frac{1}{2}v^2 = \mp\frac{32}{x}$ (+ C) or equivalent e.g. $\frac{0.1}{2}v^2 = -\frac{16}{5x}$ (+ C)	A1
	Substituting x = 2 if + used earlier or – 2 if – used in d.e. $x = 2$, $v = \pm 8 \Rightarrow 32 = -16 + C \Rightarrow C = 48$ (or value appropriate to their correct equation)	M1 A1
	$v = 0 \Rightarrow \frac{32}{x} = 48 \Rightarrow x = \frac{2}{3}$m (N.B. $-\frac{2}{3}$ is not acceptable for final answer)	M1 A1 cao **8**
	N.B $\frac{d}{dx}(\frac{1}{2}mv^2) = \frac{16}{5x^2}$, is also a valid approach. Last two method marks are independent of earlier marks and of each other	

Question Number	Scheme	Marks
3.(a)		
	Large cone small cone S	
	Vol. $\frac{1}{3}\pi(2r)^2(2h)$ $\frac{1}{3}\pi r^2 h$ $\frac{7}{3}\pi r^2 h$ (accept ratios 8 : 1 : 7)	B1
	C of M $\frac{1}{2}h$, $\frac{5}{4}h$ $\bar{x}$ (or equivalent)	B1, B1
	$\frac{8}{3}\pi r^2 h.\frac{1}{2}h - \frac{1}{3}\pi r^2 h.\frac{5}{4}h = \frac{7}{3}\pi r^2 h.\bar{x}$ or equivalent	M1
	$\rightarrow \bar{x} = \frac{11}{28}h$ *	A1 (5)
(b)	$\tan\theta = \frac{2r}{\bar{x}} = \frac{2r}{\frac{11}{28}h}, = \frac{2r}{\frac{11}{14}r} = \frac{28}{11}$	M1, A1
	$\theta \approx 686°$ or 1.20 radians	A1 (3)
	(Special case – obtains complement by using $\tan\theta = \frac{2r}{\bar{x}}$ giving 21.4° or .374 radians M1A0A0)	**8**
	Centres of mass may be measured from another point (e.g. centre of small circle, or vertex) The Method mark will then require a complete method (Moments and subtraction) to give required value for $\bar{x}$). However B marks can be awarded for correct values if the candidate makes the working clear.	

4. (a)	Energy equation with at least three terms, including K.E term $\frac{1}{2}mV^2 + ..$ $+ .. \ \frac{1}{2}.\frac{2mg}{a}.\frac{a^2}{16}, +mg.\frac{1}{2}a.\sin 30, = \frac{1}{2}.\frac{2mg}{a}.\frac{9a^2}{16}$ $\Rightarrow V = \sqrt{\frac{ga}{2}}$	M1 A1, A1, A1 dM1 A1 (6)
(b)	Using point where velocity is zero and point where string becomes slack: $\frac{1}{2}mw^2 =$ $\frac{1}{2}.\frac{2mg}{a}.\frac{9a^2}{16}, -mg.\frac{3a}{4}.\sin 30$ $\Rightarrow w = \sqrt{\frac{3ag}{8}}$ Alternative (using point of projection and point where string becomes slack): $\frac{1}{2}mw^2 - \frac{1}{2}mV_1^2, = \frac{mga}{16} - \frac{mga}{8}$ So $w = \sqrt{\frac{3ag}{8}}$	M1 A1, A1 A1 (4) M1,A1 A1 A1 **10**
	In part (a) DM1 requires EE, PE and KE to have been included in the energy equation. If sign errors lead to $V^2 = -\frac{ga}{2}$, the last two marks are M0 A0 In parts (a) and (b) A marks need to have the correct signs In part (b) for M1 need **one** KE term in energy equation of at least **3 terms** with distance $\frac{3a}{4}$ to indicate first method, and **two** KE terms in energy equation of at least **4 terms** with distance $\frac{a}{4}$ to indicate second method. SHM approach in part (b). (**Condone this method only if SHM is proved**) Using $v^2 = \omega^2(a^2 - x^2)$ with $\omega^2 = \frac{2g}{a}$ and $x = \pm\frac{a}{4}$. Using 'a' = $\frac{a}{2}$ to give $w = \sqrt{\frac{3ag}{8}}$.	M1 A1 A1 A1

5.(a)	 $\frac{mv^2}{r} = \mu N, = \mu mg$	M1, A1
	$\mu = \frac{v^2}{rg} = \frac{21^2}{75 \times 9.8} = 0.6$ *	A1 (3)
(b)	 R(↑) $R\cos\alpha, \mp 0.6R\sin\alpha = mg$	M1, A1, A1
	$\Rightarrow R\left(\frac{4}{5} - \frac{3}{5}\cdot\frac{3}{5}\right) = mg \Rightarrow R = \frac{25mg}{11}$	A1 (4)
(c)	R(←) $R\sin\alpha, \pm 0.6R\cos\alpha = \frac{mv^2}{r}$	M1, A1, A1
	$v \approx 32.5 \text{ m s}^{-1}$	dM1 A1cao (5) **12**
	In part (b) M1 needs three terms of which one is mg If cos α and sin α are interchanged in equation this is awarded M1 A0 A1 In part (c) M1 needs three terms of which one is $\frac{mv^2}{r}$ or $mr\omega^2$ If cos α and sin α are interchanged in equation this is also awarded M1 A0 A1	
	If they resolve along the plane and perpendicular to the plane in part (b), then attempt at $R - mg\cos\alpha = \frac{mv^2}{r}\sin\alpha$, and $0.6R + mg\sin\alpha = \frac{mv^2}{r}\cos\alpha$ and attempt to eliminate v	M1
	Two correct equations	A1
	Correct work to solve simultaneous equations	A1
	Answer	A1 (4)
	In part (c) Substitute R into one of the equations	M1
	Substitutes into a correct equation (earning accuracy marks in part (b))	A1
	Uses $R = \frac{25mg}{11}$ (or $\frac{25mg}{29}$)	A1
	Obtain $v = 32.5$	M1A1 (5)

6.(a)	Energy equation with two terms on RHS, $\frac{1}{2}mv^2 = \frac{1}{2}m.\frac{5ga}{2} + mga\sin\theta$	M1, A1
	$\Rightarrow v^2 = \frac{ga}{2}(5 + 4\sin\theta)$ *	A1 cso (3)
(b)	R(\\ string) $T - mg\sin\theta = \frac{mv^2}{a}$ (3 terms)	M1 A1
	$\Rightarrow T = \frac{mg}{2}(5 + 6\sin\theta)$ o.e.	A1 (3)
(c)	$T = 0 \Rightarrow \sin\theta, = -\frac{5}{6}$	M1, A1
	Has a solution, so string slack when α ≈ 236(.4)° or 4.13 radians	A1 (3)
(d)	At top of small circle, $\frac{1}{2}mv^2 = \frac{1}{2}m\frac{5ga}{2} - \frac{mga}{2}$ (M1 for energy equation with 3 terms)	M1 A1
	$\Rightarrow v^2 = \frac{3}{2}ga = 14.7a$	A1
	Resolving and using Force = $\frac{mv^2}{r}$, $T + mg = m\frac{\frac{3}{2}ga}{\frac{1}{2}a}$ (M1 needs three terms, but any v)	M1 A1
	$\Rightarrow T = 2mg$	A1 (6)
		15
	Use of $v^2 = u^2 + 2gh$ is M0 in part (a)	

7.(a)	(Measuring x from E) $2\ddot{x} = 2g - 98(x+0.2)$, and so $\ddot{x} = -49x$ SHM period with $\omega^2 = 49$ so $T = \frac{2\pi}{7}$	M1 A1, A1 d M1 A1cso (5)
(b)	Max. acceleration = 49 × max. x = 49 x 0.4 = 19.6 m s^{-2}	B1 (1)
(c)	String slack when x = –0.2: $v^2 = 49(0.4^2 - 0.2^2)$ $\Rightarrow v \approx 2.42$ m s^{-1} $= \frac{7\sqrt{3}}{5}$	M1 A1 A1 (3)
(d)	Uses $x = a\cos\omega t$ or use $x = a\sin\omega t$ but not with x = 0 or $\pm a$ Attempt complete method for finding time when string goes slack $-0.2 = 0.4\cos 7t \Rightarrow \cos 7t = -\frac{1}{2}$ $t = \frac{2\pi}{21} \approx 0.299$ s Time when string is slack $= \frac{(2)\times 2.42}{g} = \frac{2\sqrt{3}}{7} \approx 0.495$s (2 needed for A) Total time = 2 x 0.299 + 0.495 ≈ 1.09 s	M1 dM1 A1 A1 M1 A1ft A1 (7) **16**
(a)	DM1 requires the minus sign. Special case $2\ddot{x} = 2g - 98x$ is M1A1A0M0A0 $2\ddot{x} = -98x$ is M0A0A0M0A0 No use of $\ddot{x}$, just a is M1 A0,A0 then M1 A0 if otherwise correct. Quoted results are not acceptable.	
(b)	Answer must be positive and evaluated for B1	
(c)	M1 – Use correct formula with their ω, a and x but **not** x = 0. A1 Correct values but allow x = +0.2 **Alternative** It is possible to use energy instead to do this part $\frac{1}{2}mv^2 + mg \times 0.6 = \frac{\lambda \times 0.6^2}{2l}$ M1 A1	
(d)	If they use $x = a\sin\omega t$ with $x = \pm 0.2$ and add $\frac{\pi}{7}$ or $\frac{\pi}{14}$ this is dM1, A1 if done correctly If they use $x = a\cos\omega t$ with x = -0.2 this is dM1, then A1 (as in scheme) If they use $x = a\cos\omega t$ with x = +0.2 this needs *their* $\frac{\pi}{7}$ minus answer to reach dM1, then A1	